European Journal of Political Economy
Vol. 17 (2001) 157–162

European Journal of POLITICAL ECONOMY

A runoff system restores the principle of minimum differentiation

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Received 1 October 1999; received in revised form 1 April 2000; accepted 1 June 2000

Abstract

We show that the principle of minimum differentiation holds in two-round elections, for any number of candidates, regardless of the presence of entrants, or the distribution of voters' preferences. © 2001 Elsevier Science B.V. All rights reserved.

JEL classification: D72
Keywords: Runoff elections; Minimum differentiation

1. Introduction

Hotelling (1929) and Downs (1957) showed that two candidates competing in an election will both choose the position of the median voter as their platform. This result was coined the principal of minimum differentiation (MD) by Boulding (1966). Later work, however, showed that this result is highly sensitive to the assumptions made. In particular, the principle of minimum differentiation no longer holds with more than two candidates, the possibility of entry, or a change in the underlying distribution of voters' preferences. In this paper, we show that MD is restored under a runoff system, where only the two most successful candidates in the first round are allowed to run in the

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PII: S0176-2680(00)00033-1

second. Our result is robust to changes in the number of candidates, to the possibility of entry, and the distribution of voters' preferences.

In a seminal paper, Hotelling (1929) showed that two firms, in choosing a location on a straight line, will locate exactly in the middle.[1] He also suggested that this model could be applied to the case of political competition, where it implies that "each party strives to make its platform as much like the other as possible".[2] Downs (1957) studied this application in more detail. In his model, when two candidates participating in an election locate in an ideological space, both will choose a position that is equal to that of the median voter. Eaton and Lipsey (1975) relax several of the assumptions of the Hotelling/Downs model. They show that MD no longer holds when more than two candidates are competing. Notably, with three candidates, a Nash equilibrium fails to exist. These results were also proven by Selten (1971). Also, for $n > 2$, the equilibrium depends on the probability distribution of voters' positions on the line. Prescott and Visscher (1977) show that the possibility of entry will also induce candidates not to locate in the middle. Osborne (1995) concludes that "when there are more than two (potential) candidates, then the basic incentive [for MD] inherent in the Hotelling model is significantly diluted".[3] For a survey of more contributions to this literature, see e.g. Martin (1993), Chapter 10 for the economic, and Shepsle (1991) or Osborne (1995) for the political interpretation of the model.

All these results, when interpreted in a political context, assume that candidates choose a location so as to maximize the share of the vote. As the candidate with the highest share of the vote wins the election, this seems a reasonable approach. Real-world elections, however, often do not work this way. In numerous countries, such as in France, Portugal, and Austria, the winner of, for example, a presidential election is decided in two rounds, if necessary (see e.g. Lijphart, 1994). In the first round, many candidates run. In the second round, or runoff, only the two most successful candidates from the first round are allowed to compete.[4] Voters choose between these two to decide who ultimately wins the election. This system is also used in all presidential elections in Latin America (see Cox, 1997). Other countries, such as Australia, use an Alternative Vote system to elect the House of Representatives. In this system, voters already announce in the first round who they will vote for should their favorite candidate be eliminated in the first round. For more on this system, see, for example, Bogdanor (1983). It can easily be seen that the results we derive in this paper also hold for an Alternative Vote system.

[1] In fact, d'Aspremont et al. (1979) show that Hotelling's analysis is incorrect. With linear transportation costs, as Hotelling assumes, they show that two firms choose to locate at the endpoints of the interval, rather than in the middle. By competing in price, firms may also be able to attract the consumers that are located in the "backyard" of their competitor, a possibility that Hotelling does not take into account. Yet, as politicians do not set prices, this does not affect the minimum differentiation result for the case of political competition.

[2] Hotelling (1929), p. 54.

[3] Osborne (1995), p. 284.

[4] Osborne and Slivinski (1996) also study a runoff system. However, they assume citizen–candidates, i.e. each candidate uses his own preference as a platform in the election, rather than choosing a platform that maximizes the chance of winning the election.

In this paper, we show that MD is restored when elections are held under a runoff system. We show that, regardless of the number of initial candidates, all of them will choose the position of the median voter. Moreover, this result carries through in circumstances when the distribution of voters' preferences is not uniform. Finally, we show that our result also holds when potential entry is taken into account. Section 2 considers the standard case, with n candidates and a uniform distribution of voters' preferences. In Section 3 we show that entry does not change our results. Section 4 generalizes to any distribution of voters' preferences. Section 5 concludes.

2. Why a runoff system restores MD

Consider the following set-up. We have an election with two rounds. In the first round, n candidates participate. The two candidates with the highest share of the vote proceed to the second round. The candidate with the largest share of the vote in that round wins the election.

Preferences are represented by a horizontal line, normalized to [0, 1]. Before the first round, every candidate i chooses his position P_i. We assume that this position cannot be changed between rounds, for example since such a shift in position would undermine the candidate's credibility, destroying his chances to win the election. Voters are uniformly distributed on [0, 1], an assumption we will relax in Section 4. Voters always vote for the candidate with the position that is closest to theirs, i.e. there is sincere voting. In case of a tie, they decide randomly which candidate to vote for. We can now establish the following result.

Theorem 1. *When voters' preferences are uniformly distributed, the unique symmetric Nash equilibrium has all candidates choosing the position of the median voter:* $P_i = (1/2)\ \forall_i$.

Proof of Theorem 1. Since the case $n = 2$ is the standard one-round case already considered by Hotelling (1929), we restrict attention to $n > 2$. First assume that all candidates $i = 1, \ldots, n$ set $P_i = 1/2$. Now suppose that candidate n considers a defection to some $P'_n \neq 1/2$. Without loss of generality, suppose $P'_n < 1/2$. The share of the vote of candidate n in the first round of the election then equals $S_n^1 = P'_n + ((1/2) - P'_n)/2 = (1/4) + (1/2)P'_n$. The other $n-1$ candidates each have a share $S_i^1 = \{(1/2) + ((1/2) - P'_n)/2\}/(n-1) = ((3/4) - (P'_n/2))/(n-1)$. Candidate n will reach the second round with certainty whenever $S_n^1 > S_i^1$, thus if

$$(n-1)(1 + 2P'_n) > 3 - 2P'_n, \tag{1}$$

or

$$P'_n > \frac{4-n}{2n}. \tag{2}$$

With $n = 3$, this condition is satisfied for $P_n' > 1/6$, Any defection with $P_n' < 1/6$ then implies that candidate n already drops out in the first round. With $n = 4$, it is satisfied for $P_n' > 0$. For larger n, it always holds. However, when candidate n follows such a defection, he will face a runoff with some candidate j who has $P_j = 1/2$. He will always lose this runoff: his share of the vote will equal $S_n^2 = (1/2)P_n' + (1/4)$, whereas that of his competitor equals $S_i^2 = (3/4) - (1/2)P_n'$, which, with $P_n' < 1/2$, is always higher. Thus, defecting from an equilibrium with $P_i = (1/2)\ \forall_i$, is never profitable. The uniqueness of the symmetric Nash equilibrium is trivial. Suppose there is a unique symmetric Nash equilibrium with $P_i \neq 1/2$, and equal for all i. Then any candidate j can improve by defecting to $P_j = 1/2$. By doing so, he wins both the first and second round. □

The above proof also provides an intuition for this result. By choosing a position different from that of the median voter, a candidate may win the first round. In the second round, however, he will be beaten by one of the candidates that did choose the median voter's position.[5]

3. The case of potential entry

Prescott and Visscher (1977) argue that MD no longer holds when candidates consider the possibility of entry of third candidates. When two incumbents do choose the median position, they argue, such an entrant can secure almost half of the vote by entering just to the left or just to the right of the median voter, leaving the original incumbents with only $1/4$ of the vote. When incumbents anticipate this possibility, they will not locate at the median voter but, rather, at positions $1/4$ and $3/4$.

In our model, however, this argument does not hold. Suppose that n candidates are located at $1/2$, and a new candidate enters. The best the new candidate can do is locate at $1/2$ as well, by the same argument as in the Proof of Theorem 1. Entering at any other location will result in losing the election in the second round. Hence, the incumbents do not have an incentive to locate differently. We thus have

Theorem 2. *With potential entry, the unique symmetric Nash equilibrium has all candidates choosing the position of the median voter.*

[5] Admittedly, the equilibrium described in the theorem is not the unique Nash equilibrium. For example, with $n = 4$, the same argument as in the proof can be used to show that $P_1 = P_2 = (1/2) - \varepsilon$, and $P_3 = P_4 = (1/2) + \varepsilon$ is also an equilibrium, for small enough ε. Yet, such an equilibrium does not exist for all n. With n odd, it can be shown that no other Nash equilibrium exists. Also, we believe that the equilibrium in the text is the most natural one. In any non-symmetric equilibrium, some coordination mechanism is needed to determine which candidate chooses which position. Such problems do not exist in our equilibrium. Therefore, we believe our equilibrium is focal in the sense of Schelling (1960).

4. The case of a general distribution function

So far, we have restricted attention to the case in which the preferences of the voters are uniformly distributed. Eaton and Lipsey (1975) show that, with $n > 2$, the equilibrium depends on the distribution of voters. In this section, we show that, in our set-up, this distribution does not affect the results.

Theorem 3. *For any continuous distribution of voters' preferences, the unique symmetric Nash equilibrium has all candidates choosing the position of the median voter:* $P_i = p_m \ \forall i$.

Proof of Theorem 3. The proof is virtually identical to that of Theorem 1. Suppose voters' preferences are described by a probability density function $F(p)$, with $F(0) = 0$ and $F(1) = 1$. The median voter p_m is the one for whom $F(p_m) = 1/2$. Restrict attention to the case $n > 2$, the case $n = 2$ being the standard one-round case already proven by Eaton and Lipsey (1975). Assume that all candidates $i = 1, \dots, n$ set $P_i = p_m$. Now suppose that candidate n considers a defection to some $P_n' \neq p_m$. Without loss of generality, suppose $P_n' < p_m$. The share of the vote of candidate n in the first round of the election then equals $S_n^1 = F((P_n' + p_m)/2)$. The other $n - 1$ candidates have a share $S_i^1 = \{1 - F(P_n' + p_m)/2)\}/(n - 1)$. Candidate n will reach the second round with certainty whenever $S_n^1 > S_i^1$, thus if $F((P_n' + p_m)/2) > (1/n)$. This is intuitive; it implies that the share of the vote of candidate n is higher than the average share of the vote of all n voters. Suppose n chooses a defection such that this condition is met. Then, in the second round, he will face a runoff with some candidate j with $P_j = p_m$. He will always lose this runoff. □

5. Conclusion

It is well established that two candidates that try to win an election will both choose a location in ideological space that coincides with that of the median voter. Yet, a wealth of literature suggests that this result is not robust. In particular, this principal of minimum differentiation does not hold with more than two candidates, when the distribution of voters' preferences is not uniform or if there is potential entry. Yet, this literature assumes that elections are held in only one round, and that the candidate with the highest share of the vote wins the election. Many real-world elections do not work this way. Often, an election consists of two rounds. In the first round, all candidates participate. In the second round, only the two candidates with the best result in the first round can run. In this paper, we showed that, in such elections, the principle of minimum differentiation again holds. Moreover, this result is robust with respect to the number of candidates, the distribution of voters' preferences and potential entry. Therefore, a runoff system restores the principle of minimum differentiation, regardless of the exact specification of the model.

Acknowledgements

The authors thank Jakob de Haan, Peter Kooreman, Lambert Schoonbeek, and Bert Smid for useful comments.

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