Advertising for Attention in a Consumer Search Model*

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Abstract

We model the idea that when consumers search for products, they first visit the firm whose advertising is more salient. The gains a firm derives from being visited early increase in search costs, so equilibrium advertising increases as search costs rise. As a result, higher search costs may decrease both consumer welfare and firm profits. We extend the basic model by allowing for firm heterogeneity in advertising costs. Firms that raise attention more easily advertise more but also charge lower prices and obtain higher profits. As advertising cost asymmetries increase, consumer surplus falls and aggregate profits rise.

JEL Classification: D83, L13, M37.

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1 Introduction

Advertising is an important part of economic activity. According to PriceWaterhouseCoopers (2005) worldwide advertising in 2005 amounted to a staggering $385 billion. The outcome of such an impressive investment is that the average citizen is exposed to hundreds of commercial messages every day.[1] However, very few of these messages are able raise the attention of consumers.[2]

Part of the empirical marketing literature on advertising indeed acknowledges that an important role of advertising is to create firm *saliency*, that is, the prominence of a brand or a shop in consumers' memories.[3] The economics literature, nicely surveyed in Bagwell (2007), has focused on other roles of advertising. Most papers study either persuasive advertising that directly affects consumers' willingness to pay, or informative advertising that informs consumers about the availability, characteristics or price of a product. To the best of our knowledge the economics of saliency enhancing advertising, i.e. advertising that increases shop or brand memorability, have not yet been studied. This paper tries to fill that gap.

In our model, firms sell differentiated products and consumers search shops to find a product to their liking. A consumer buys a product if she finds a deal that is attractive enough to make it not worth her while to continue searching. It is thus in the interest of firms to be visited earlier than the rivals, which gives them an incentive to invest in saliency. We assume that the order in which firms are sampled is influenced by how much they advertise. A firm that advertises more is remembered more readily and hence also more likely to be visited. Although advertising does not directly increase willingness to pay, in equilibrium consumers do increase their propensity to buy a product when they see it advertised.

We thus provide a framework to understand the effects of search costs on saliency enhancing advertising, product prices and firm profits. We find that both price and advertising expenditures increase in search costs. As search costs increase, consumers are more reluctant to visit

[1]Estimates of this number vary widely (see e.g. http://www.hhcc.com/?p=468 for a discussion). In 1972, Britt et al. (1972) already find between 300 and 600 messages per day. Arens et al. (2007, p.6) even claims that "as a consumer, you are exposed to hundreds and maybe even thousands of commercial messages every day."

[2]Franz (1986), for example, reports that out of more than 13,000 individuals asked in 1985 about the advertisements (ads) that were seen, heard or read in the past 30 days, 53 % were unable to remember any specific one.

[3]For example, Unnava and Burnkrant (1991) study the effects of advertising repetition on consumer memorability of a brand or shop; Burke and Srull (1988) and Alba and Chattopadhyay (1986) study the existence of saliency enhancing advertising externalities and point out that investments in "salience" of one firm *inhibit* the recall of alternative firms. Danaher et al. (2008) show that a competitor's advertising adversely affects a brand's advertising effectiveness in terms of sales.

many shops. This has two effects. First, a typical shop has more market power over consumers that do pay a visit, hence it charges a higher price. Second, it becomes more important for a firm to be salient and to be visited early, hence firms advertise more. The effect of an increase in search costs on firm profitability is ambiguous. If search costs are relatively small, the price effect dominates and equilibrium profits increase with search costs. If search costs are relatively high, the price effect may be more than offset by the rent-dissipation effect of increased advertising, and equilibrium profits decrease with search costs. Our model thus provides an instance in which firms do not necessarily benefit from higher search costs, contrary to received wisdom in the literature (see e.g. Anderson and Renault, 1999; Wolinsky, 1986). In fact, with search costs high enough, profits may even be lower than in a frictionless world without search costs.

In our basic model, firms find themselves in a classic prisoners' dilemma. If a firm advertised less than the others, it would be more likely to be pushed to the end of consumers' search order. In equilibrium all firms advertise with the same intensity so consumers recall each firm with the same probability. From a welfare point of view, advertising is purely wasteful so an advertising ban would be desirable.[4]

To analyze the effect of asymmetries, we also study a duopoly where one firm is more efficient in generating saliency. We find that this firm advertises more and hence attracts a larger share of consumers on their first visit. This firm also charges a lower price. Consumers that choose to visit a second firm reveal that they do no particularly like the product the first firm offered. Hence, these consumers are less price-sensitive at the second firm that they visit. As the less efficient firm attracts relatively more of these consumers, it charges a higher price. Still, equilibrium profits of the more efficient firm are higher. Advertising now has value for consumers as it helps them channel their first-visits towards better deals. Nevertheless, because of the higher price charged by the less efficient firm, total consumer surplus is lower than when firms are visited randomly. Savings in advertising outweigh consumer losses and overall welfare increases with cost asymmetries.[5]

In sum, across industries, our model predicts a positive correlation between search costs, advertising expenditures and prices. Firms in an industry with higher search costs set higher

[4]Of course, if we take into account the advertising industry and we view advertising outlays as mere transfers from producers to advertisers, an advertising ban would be inconsequential.

[5]The welfare result is due to cuts in advertising expenditures. Therefore, if we took into consideration the fact that advertising outlays are not lost but transferred to the advertising industry, then overall welfare would be lower than if advertising were banned altogether.

prices, but also advertise more. Within an industry, we predict a negative correlation between prices on the one hand, and advertising expenditures and market shares on the other. Firms that are more efficient in generating saliency attract more consumers, set lower prices and make higher profits. Other papers also predict a negative relationship between advertising and prices, but for different reasons. In Robert and Stahl (1993) firms can advertise prices on a search market with homogeneous products, and advertise lower prices more intensively. In Bagwell and Ramey (1994) advertising is used as a coordination device for firms to attract more consumers and hence have lower costs, allowing them to charge lower prices. In our model, firms that advertise more attract a pool of consumers that is more price sensitive, and therefore charge lower prices.

We assume that the order in which firms are visited is influenced by advertising efforts. In an empirical study, Hortaçsu and Syverson (2004), use advertising as one proxy for the sampling probability of a mutual fund. Our model is consistent with that idea. Most theoretical papers in the search literature assume that consumers sample firms randomly,[6] but there are exceptions. In Chen and He (2006) and Athey and Ellison (2009) firms pay for placing ads on a search engine. A firm has private information about its quality, i.e. the probability that consumers like its product. In equilibrium, higher placed ads represent higher quality products and consumers rationally search firms in the order in which their ads are placed. In Arbatskaya (2007), search order is exogenously given. Prices fall in search order: a consumer that walks away from a firm reveals that she has low search costs, giving the next firm an incentive to charge a lower price. Zhou (2010) studies a similar model with differentiated products but finds the opposite effect on price. The intuition is similar to our model: a consumer that walks away has fewer options left and is thus less price sensitive. In Wilson (2010) a firm can choose the size of search cost consumers have to incur to visit it. Consumers are then more likely to first visit firms with low search cost, and prices fall in search order. Finally, and more directly related to our specific model, Armstrong et al. (2009) study a search market with differentiated products where one firm is always visited first, while the other firms are sampled randomly if a consumer decides not to buy from the prominent firm. Indeed, our model can be interpreted as one in which firms invest in prominence but where prominence can only be imperfect.

Our model is related to several other literatures. One is that on information overload. In Van Zandt (2004), consumers can only process a fixed and finite number of advertising messages. They pick messages at random to process, consistent with our model. Anderson de

[6]See e.g. the seminal contributions of Burdett and Judd (1983) and Wolinsky (1986).

Palma (2009) extend van Zandt (2004) by endogenizing the decisions of consumers. Work that looks at the relation between advertising and match values include Meurer and Stahl (1994) , where firms can advertise match values, and Johnson and Myatt (2006), where advertising affects the distribution of match values. In Johnson (2009) firms can use ads to send signals about match values, but consumers can choose to block those ads. Finally, Loginova (2009) describes the market equilibrium in a model where consumers have an imperfect memory for the ads that they have seen.

The remainder of the paper is structured as follows. In section 2 we describe the set-up of the model. The equilibrium results for symmetric firms are derived in subsection 3, and the comparative statics results on the effects of search costs on advertising efforts, prices and profits are given in subsection 3.3. Section 4 presents results for a market with asymmetric firms. Section 5 concludes. The proofs of the main results are placed in the Appendix.[7]

2 The model

Consider a market where n firms sell horizontally differentiated products. Marginal costs are constant and normalized to zero. For simplicity and without loss of generality, we assume that there is one consumer. She derives utility

$$u^i(p_i) = \mu\varepsilon_i - p_i,$$

if she buys product i at price p_i, $i = 1, 2, ..., n$. We assume that ε_i is the realization of a random variable with distribution F and continuously differentiable log-concave density f with support normalized to $[0, 1]$. The scalar μ is a parameter that captures the amount of heterogeneity in consumer's tastes: the higher μ, the more differentiated products are in the perception of the consumer. The product $\mu\varepsilon_i$ can then be viewed as a match value between the consumer and the good sold by firm i. Match values are independently distributed across products. No firm can observe ε_i so practising price discrimination is not feasible. The consumer only learns ε_i upon visiting firm i. We denote the monopoly price by p^m, i.e., $p^m = \arg\max_p\{p(1-F(p/\mu))\}$.

The consumer must incur a search cost s in order to visit a firm i and learn both its price p_i and her match value ε_i. We assume the consumer searches sequentially with costless recall. The search cost s is assumed to be relatively small, in particular,

$$0 \leq s \leq \overline{s} \equiv \int_{p^m/\mu}^{1} (\mu\varepsilon - p^m) f(\varepsilon)d\varepsilon.$$

[7] Extended proofs containing all the details of the derivations are included in an online Technical Appendix, available at the website of the journal.

This implies that, even if all firms charged the monopoly price, the consumer would still be willing to conduct a first search.

Firms engage in an advertising battle to lure consumers to their shops. In particular, we assume that at any moment during the search process, a consumer is more likely to go to a firm i if she has had more exposure to the ads of that firm (or if its ads have happened to be relatively more salient than other firms' ads).[8]

Let a_i denote the number of ads of a firm i. The cost of producing a_i ads is $\phi_i(a_i)$, with $\phi_i' > 0$ and $\phi_i'' \geq 0$. Given an advertising strategy profile $(a_1, a_2, ..., a_n)$, suppose that the consumer has already visited v firms. Let N denote the set of firms and $V \subset N$ the set of visited firms. We assume that the probability that the consumer will recall firm $i \in N \backslash V$ in her next search is given by[9]

$$\frac{a_i}{\sum_{j \in N \backslash V} a_j}.$$

We thus model advertising as an attention-seeking contest in the spirit of Tullock (1980).[10] As opposed to the standard Butter's (1977) technology, this modelling of the recall probability captures the *inhibition* effects that own advertising appears to have on the recall of competing brands (cf. Burke and Srull, 1988; Alba and Chattopadhyay, 1986).[11] A firm that does not advertise at all is visited last. If no firm advertises, the consumer visits firms randomly.

Intuitively, one can think of each advertisement of a firm as a ball this firm puts in an urn. Each firm can put as many balls in the urn as it likes. Whenever the consumer needs a product, she draws one ball from the urn and visits the corresponding firm. If, after the first visit, the consumer decides to visit another firm, she proceeds in the same way: again draw a ball from the urn and visit the corresponding firm provided it has not been visited yet; and so forth.[12] We assume that the consumer cannot observe the advertising efforts of the firms.

[8] In the marketing and business literatures, the ease with which a brand/shop comes to mind is referred to as "top-of-mind awareness" (see e.g. Kotler, 2000).

[9] Alternatively, we may assume that firms decide on the amount A_i to spend on advertising, while the amount of ads of firm i is given by $a_i = \tau_i(A_i)$. It is easy to see that this specification yields the exact same outcomes if we choose $\phi_i(\cdot) = \tau_i^{-1}(\cdot)$. Skaperdas (1996) shows that any function that satisfies a number of basic properties necessarily has the form proposed here, with $\tau_i(A_i) = \alpha A_i^{\gamma}$, for some $\alpha, \gamma > 0$. See also Kooreman and Schoonbeek (1997).

[10] The theory of contests is extensive (for a survey, see Lockard and Tullock, 2001). In most of this literature, agents' valuations of the prize are independent of their efforts. Schmalensee (1976) uses a similar idea in the context of advertising, but in his model prices are exogenously given. Hortaçsu and Syverson (2004), in their empirical study of price dispersion in the mutual fund industry, also model the funds' sampling probabilities in a similar way. Chioveanu (2008) also uses this type of advertising technology in the context of persuasive (loyalty-inducing) advertising in a market for homogeneous products.

[11] Under Butters' technology the fraction of consumers who are informed about firm i's product is independent of the rival firms' advertising efforts.

[12] An alternative formulation would have firms engaged in an advertising race for consumers' attention (akin

One could argue that the way in which consumers react to advertising in our model is rather ad-hoc. Still, we believe it makes sense. As argued in the introduction, consumers often do not deliberately pay attention to ads, and it is impossible to process all ad messages that are received. The more often consumers are exposed to an ad, however, the more likely they are to remember it, and the more likely they are to act upon it, as the marketing literature also suggests. That is what we try to capture with our model.

The timing in our model is as follows. First, firms simultaneously decide on advertising and prices. Second, consumers sequentially search for a satisfactory deal following the recall process described above. In Section 3.1 we focus on the case where all firms have the same advertising technology. In Section 4 we study a market where firms differ in their advertising technologies.

3 Symmetric firms

3.1 Analysis

In this section we assume that all firms have the same advertising technology, i.e. $\phi_i = \phi$ for all i, and look for a symmetric Nash equilibrium (SNE). Consider firm i. Suppose that all firms different from i charge price p^* and set advertising level a^*. A SNE then requires that the best reply of firm i is to also set (a^*, p^*). To calculate firm i's payoff, we need to take into account the order in which firm i may be visited, and the probability it makes a sale conditional on being visited.

Suppose that the buyer approaches firm i in her first search. This firm provides her with net utility $\mu\varepsilon_i - p_i$. If $\mu\varepsilon_i - p_i < 0$, the consumer will search again. Suppose $\mu\varepsilon_i - p_i \geq 0$. A visit to some other firm j will give her utility $\mu\varepsilon_j - p^*$. This is higher than the utility from buying from firm i if $\varepsilon_j > \varepsilon_i - \Delta$, with $\Delta = (p_i - p^*)/\mu \geq 0$. If we define $x \equiv \varepsilon_i - \Delta$, the expected benefit from searching once more equals $\mu g(x)$ where

$$g(x) \equiv \int_x^1 (\varepsilon - x) f(\varepsilon) d\varepsilon.$$

An additional search is worthwhile if and only if these incremental benefits exceed the cost of search s. The buyer is exactly indifferent between an additional search and accepting the offer at hand if $x \geq \hat{x}$, with $\hat{x}$ implicitly defined by

$$g(\hat{x}) = \frac{s}{\mu}. \tag{1}$$

to the patent race models in the R&D literature). The results in Baye and Hoppe (2003) show that these two formulations are strategically equivalent.

The function g is monotonically decreasing. Moreover, $g(0) = E[\varepsilon]$ and $g(1) = 0$. It is readily seen that $\overline{s} < \mu E[\varepsilon]$. Therefore, for any $s \in [0, \overline{s}]$, there exists a unique $\hat{x} \in [p^m/\mu, 1]$ that solves (1).

Since any equilibrium necessarily has $\hat{x} \geq p^*/\mu$, the probability that a buyer stops searching at firm i given that firm i is visited, is equal to

$$\Pr[x > \hat{x} \text{ and } \mu\varepsilon_i > p_i] = \Pr[x > \hat{x}] = 1 - F(\hat{x} + \Delta),$$

provided that the deviation is sufficiently small. If we denote the probability that a consumer visits firm i in her first search and buys there right away as $\lambda_1^i(a_i, p_i; a^*, p^*)$ we have:

$$\lambda_1^i(a_i, p_i; a^*, p^*) = \frac{a_i}{a_i + (n-1)a^*}(1 - F(\hat{x} + \Delta)) \tag{2}$$

Now consider the case in which a consumer goes to firm i in her k^{th} search, $k = 2, 3, ..., n$ and then decides to buy there. This means that the consumer has visited $k - 1$ other firms first but decided not to buy there. In SNE, whenever she walks away from a firm, she expects price to be equal to p^* in the next shop. The probability that she walks away from a rival firm is thus $F(\hat{x})$. If we denote by $\lambda_k^i(a_i, p_i; a^*, p^*)$ the probability that firm i is visited in k^{th} place and the consumer decides to buy from i right away, we have

$$\lambda_k^i(a_i, p_i; a^*, p^*) = \frac{a_i}{a_i + (n-k)\,a^*} \prod_{\ell=1}^{k-1} \frac{(n-\ell)\,a^*}{a_i + (n-\ell)\,a^*} F(\hat{x})^{k-1} \left[1 - F(\hat{x} + \Delta)\right]. \tag{3}$$

where we have used the fact that the reservation value $\hat{x}$ is the same no matter how many firms the consumer has already visited (see Kohn and Shavell, 1974).

There is a probability that the consumer initially decides to walk away from firm i only to find that, after having visited all firms in the industry, firm i offered the best deal after all. Of course, the consumer will then return to firm i to buy there. The probability of this occurring is

$$\Pr[\max\{x, \max_{j \neq i}\{\varepsilon_j\}\} < \hat{x} \text{ and } \mu\varepsilon_i - p_i > \max_{j \neq i}\{\mu\varepsilon_j - p^*\} \text{ and } \mu\varepsilon_i > p_i]$$

This probability is independent of the order in which firms are visited and will be denoted as

$$R(p_i; p^*) = \int_{p_i/\mu}^{\hat{x}+\Delta} F(\varepsilon - \Delta)^{n-1} f(\varepsilon) d\varepsilon. \tag{4}$$

For p_i close to p^*, we can now write firm i's expected profits as

$$\Pi_i(a_i, p_i; a^*, p^*) = p_i \left[\sum_{k=1}^{n} \lambda_k^i(a_i, p_i; a^*, p^*) + R(p_i; p^*)\right] - \phi(a_i). \tag{5}$$

Note that this expression is only valid for small deviations from the tentative equilibrium. For large deviations such that $F(\hat{x}+\Delta)=1$ the profit function looks different as the consumer would always walk away from firm i, perhaps ultimately coming back to buy from it. We take this case into account in the proof of Proposition 1.[13]

Upon observing the payoff formulation in (5), it is clear our game differs from standard rent-seeking games in two important respects. First, advertising is not a winner-takes-all contest since after the consumer has visited a firm, she may still decide to go to a different one. Second, because consumer search and prices are endogenous, the prizes obtained by the winner, second winner, etc., all the way to the loser of the advertising contest depend in an intricate way on the firms' advertising efforts and prices.

Proposition 1 *If a SNE exists, advertising levels and prices are given by the following system of equations:*

$$a^*\phi'(a^*) - \frac{p^*}{n}\left(1 - F(\hat{x})^n - \sum_{k=0}^{n-1} \frac{F(\hat{x})^k\left(1 - F(\hat{x})^{n-k}\right)}{n-k}\right) = 0 \tag{6}$$

$$\frac{1 - F(p^*/\mu)^n}{np^*/\mu} - \frac{f(\hat{x})}{n}\frac{1 - F(\hat{x})^n}{1 - F(\hat{x})} + \int_{p^*/\mu}^{\hat{x}} F(\varepsilon)^{n-1} f'(\varepsilon)d\varepsilon = 0 \tag{7}$$

Suppose that F represents the uniform distribution and that ϕ'' is sufficiently large. Then a symmetric equilibrium exists and is unique.

The proof of this result proceeds along the following lines. First, we show that the FOCs imply the expressions (6) and (7). Second, we prove that a solution to the system of equations (6) and (7) exists and is unique if we assume $f' \geq 0$. Third, we show that when F is the uniform distribution and ϕ'' is large, then a firm i's payoff function (5) is globally concave.[14] Finally, we prove that large deviations from (p^*, a^*), for which the profit function (5) is no longer the relevant one, are not profitable either. These steps establish the result.

Example. To illustrate the analysis, we provide a simple example. Suppose that we have two firms, match values are uniformly distributed and advertising costs are given by the polynomial function $\phi(a) = \alpha a^\gamma$, $\gamma \geq 1$. Suppose firm 2 charges p^* and advertises with

[13]Similarly, if a firm were to set an advertising effort arbitrarily close to zero, then the firm would be visited last with a probability arbitrarily close to one and then the payoff would be similar. In the Appendix we show that these cases are not really problematic.

[14]For general distributions functions F, however, the expression $p_iR(p_i;p^*)$ may not be quasi-concave in p_i and so the profit $\Pi_i(\cdot)$ may be maximized at a pair other than (p^*, a^*). Under log-concavity of the density function f, the expression $p_iR(p_i;p^*)$ is quasi-concave in p_i (see Caplin and Nalebuff, 1991) but this does not guarantee that $\Pi_i(\cdot)$ is also quasi-concave in p_i (the sum of quasi-concave functions need not be quasi-concave).

intensity a^*. We look for a best reply for firm 1. Suppose firm 1 uses strategy (p_1, a_1). Following the discussion above, if firm 1 is visited first, which occurs with probability $a_1/(a_1+a_2)$, the consumer will buy from firm 1 straight away with probability $1-F(\hat{x}+\Delta)$, which is this case equals $1-\hat{x}-(p_1-p^*)/\mu$. If firm 1 is visited second, which occurs with the remaining probability $a_2/(a_1+a_2)$, the consumer will walk away from firm 2 and buy immediately from firm 1 with probability $F(\hat{x})(1-F(\hat{x}+\Delta))$, which equals $\hat{x}(1-\hat{x}-(p_1-p^*)/\mu)$ in this example. The probability that a consumer initially turns down all offers but returns to firm 1 is given by (4), which in this case equals $(\hat{x}^2-(p^*/\mu)^2)/2$. Firm 1's profit thus equal

$$\Pi_1 = p_1\left(\frac{a_1+a^*\hat{x}}{a_1+a^*}\left(1-\hat{x}-\frac{p_1-p^*}{\mu}\right)+\frac{1}{2}\left(\hat{x}^2-\frac{{p^*}^2}{\mu^2}\right)\right)-\alpha a_1^\gamma$$

Taking the FOCs and imposing symmetry yields

$$p^* = \frac{\mu}{2}\left(\sqrt{5+\hat{x}(2+\hat{x})}-1-\hat{x}\right) \quad (8)$$

$$a^* = \left(\frac{p^*(1-\hat{x})^2}{4\alpha\gamma}\right)^{1/\gamma}. \quad (9)$$

where $\hat{x}$ follows from $\int_{\hat{x}}^1(\varepsilon-x)\,d\varepsilon = s/\mu$ so $\hat{x}=1-\sqrt{2s/\mu}$.

3.2 Comparative Statics

Proposition 2 *The comparative statics of the SNE described in Proposition 1 are as follows:*

1. *An increase in the marginal cost of advertising has no effect on equilibrium price p^* and lowers the equilibrium number of ads a^*.*

2. *If the density of match values is non-decreasing, $f'\geq 0$, an increase in search cost s raises both the equilibrium price p^* and the equilibrium number of ads a^*.*

3. *For μ sufficiently large, an increase in μ raises equilibrium price p^*. When $n=2$ and F is the uniform distribution, the equilibrium number of ads a^* decreases in μ.*

4. *As the number of firms approaches infinity, per-firm advertising goes to zero and aggregate industry advertising converges to the constant $\mu(1-F(\hat{x}))/f(\hat{x})\phi'(0)$. If the number of firms is sufficiently low and match values are uniformly distributed, an increase in n increases per-firm advertising effort.*

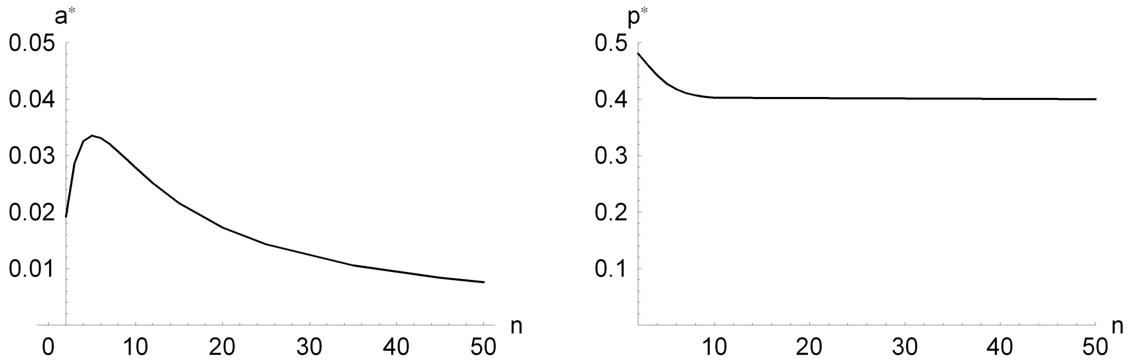
Since all firms advertise with the same intensity in a SNE, changes in advertising costs have no effect on equilibrium prices; these are only affected by relative differences in advertising levels.[15] Naturally, if advertising is more expensive, firms use less of it.

The result on the relationship between prices and search costs is similar to Anderson and Renault (1999) who study a setting where in equilibrium every consumer buys a product so the market is always fully covered. Our result extends theirs to a setting where industry demand is elastic. As search costs increase, the probability that a consumer walks away from a firm to sample another one decreases. This confers market power to a firms that is visited and hence prices increase. The result on the relationship between search costs and advertising is novel. An increase in search costs increases the market power of a firm that is visited. Hence it becomes more desirable for a firm to attract the consumer. As a result, firms advertise with greater intensity as search costs rise.

As products are more differentiated, consumers are more likely to search. This makes it less urgent for firms to be visited first, which implies that they have less of an incentive to advertise. At the same time, more product differentiation may also imply higher prices, which would give firms an incentive to advertise more. In the case of a uniform distribution and two firms, the first effect dominates. The ambiguous effect of product differentiation on prices is comparable to that in Anderson and Renault (1999).

An increase in the number of firms has two effects on firms' incentive to advertise. First, if there are more firms that put out ads, the marginal effectiveness of an additional ad decreases. This lowers the incentive to advertise. Second, as the number of firms increases, it becomes more important to attract a consumer early on. This raises the incentive to advertise. If the number of firms is small, the second effect dominates. With many firms, the first effect does. As the number of firms goes to infinity, a firm's demand converges to zero so equilibrium advertising levels also go to zero. Aggregate advertising converges though to $\mu(1-F(\hat{x}))/f(\hat{x})\phi'(0)$. Figure 1 shows that advertising intensity is non-monotonic in the number of firms for the case where F is the uniform distribution. Prices and profits of the firms also decrease in n.

[15]The fact that advertising costs do not affect equilibrium prices is an artifact of the symmetry of equilibrium. Later in Section 4 we shall for example see how lowering the advertising cost of a firm results in a fall in its equilibrium price.



(a) Advertising intensity

(b) Price

Figure 1: Price, ad intensity and the number of firms ($\mu = 1$, $f = 1$, $\phi(a) = a$).

3.3 Profits and welfare

Search costs are generally seen as a boon to firms. As search costs increase, firms have more market power, which leads to higher profits (see e.g. Anderson and Renault, 1999; Wolinsky, 1986). The following result however shows that this is not necessarily true in our model.

Proposition 3 *Assume the density of match values f' is non-decreasing. Then:*

1. *The profits of a firm increase in search cost s if the search cost is small enough.*
2. *Consider the family of advertising cost functions $\phi(a) = \alpha a^\gamma$ with $\alpha > 0, \gamma \geq 1$. Then (i) the profits of a firm increase in search cost s if γ is sufficiently large. (ii) For γ small and sufficiently large search cost s, profits may decrease in s and eventually fall below the profits that firms would make in a frictionless world. In particular, this is true with 2 firms, uniformly distributed matching values and linear or quadratic advertising costs.*

An increase in search cost s has two effects on firm profits that go in opposite direction. With an increase in s, firms gain market power over customers that pay them a visit, which allows them to charge a higher price. This tends to increase profits. But this also implies that it becomes more attractive for each individual firm to invest in saliency and try to beat its rivals in the battle for consumer attention. As a result, firms spend more on advertising, which tends to lower firm profits. When search costs are small, the price effect dominates and firms gain from an increase in search costs. When search costs are large, the advertising effect may dominate and profits may then decrease with higher search costs.[16] Interestingly, we may even have an over-dissipation of the incremental rents generated by greater market power in the sense that firms spend more resources to capture the additional rents than those rents are actually worth. This effect can become so severe that firms end up obtaining profits that are lower than those in a world with zero search costs. In Figure 2 we plot equilibrium profits against search costs for the advertising cost function $\phi(a) = \alpha a^{\gamma}$ for different values of γ. The dashed line shows the profits firms would make if search costs were zero ($s = 0$).

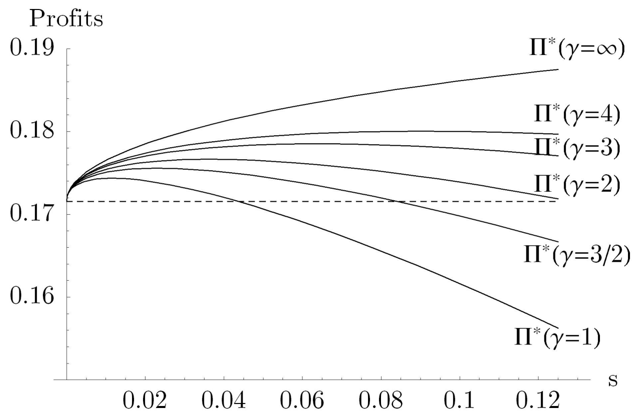


Figure 2: Equilibrium profits ($n = 2$, $f = 1$, $\phi(a) = \alpha a^{\gamma}$).

Interestingly, when search costs are sufficiently high, a fall in search costs may result in a Pareto improvement: consumers are better off as equilibrium prices decrease, while firms are better off as equilibrium profits increase.[17] When search costs are low instead, a decrease in search costs favors consumers at the expense of firms.

Note that, in this model, firms find themselves in a prisoners' dilemma. If a firm advertised less than the rest, the chance that this firm is pushed to the end of consumers search order would be higher. In equilibrium all firms advertise with the same intensity, which implies that consumers end up recalling each firm with the same probability. Firms would thus be better off if advertising were banned, while consumers would not be affected. From a welfare point

[16]It can be shown that this is not only true in the case described in the Proposition. It can be shown that profits will always decrease with high enough search costs if the market is fully covered, as in Anderson and Renault (1999), or if the first search is costless, regardless of the number of firms and the distribution of matching values. Details are available from the authors upon request.

[17]Of course, here we are not taking into consideration the advertising industry. If we did, advertisers would lose as search costs fall (advertising expenditures are just transfers from the product market to the advertising industry).

of view, advertising is purely wasteful.[18]

4 Asymmetric firms

4.1 Introduction

The analysis in the previous section has ex-ante symmetric firms. This implies that in equilibrium all firms that have not yet been visited are always equally likely to be visited next. This is no longer true if we allow for asymmetries across firms. In such a case, advertising efforts are expected to differ across firms, which implies that some firms will more likely be visited first than others, which in turn affects firms' pricing incentives. In this section we ask: Do firms that attract more consumer first-visits charge higher or lower prices? More specifically, is higher advertising correlated with higher prices, or with lower ones? How are consumer welfare and firm profits affected by firms' asymmetries?

There are alternative ways to introduce asymmetries across firms. For example, firms could face different advertising costs, have different marginal costs of production or offer products with different quality. To focus on a case where the asymmetry in equilibrium prices stems *exclusively* from differences in advertising levels, we assume that firms differ in the costs they have to incur to undertake an advertising campaign. Technically, we write $\phi_i(a_i)$, $i = 1, 2$.[19] Introducing asymmetries complicates the analysis significantly. We therefore have to restrict ourselves to a setting with 2 firms and a uniform distribution of matching values. We will also set $\mu = 1$ in what follows. Even in such a simple setting it is difficult to derive analytical results, so we will partly have to resort to a numerical analysis.

One complication has to do with consumer search behavior after out-of-equilibrium moves. Suppose that firms charge different prices in equilibrium. Let us assume consumers know the equilibrium prices but do not know which firm has which price. Suppose now that a consumer observes an out-of-equilibrium price at her first visit. Her decision whether to continue searching will then be affected by whether she interprets this out-of-equilibrium price as coming from the low-price firm or from the high-price firm.

[18] Advertising does not add value in the product market. If we take into account the advertising industry and view advertising efforts as transfers from producers to advertisers, an advertising ban would be inconsequential.

[19] Notice that in this case, the equilibrium is still symmetric in the absence of advertising. If we for example chose marginal costs to be different, say $mc_1 = r > 0$ and $mc_2 = 0$, price variation due to marginal cost differences would be augmented by price variation due to different advertising intensities. In any case, the results we obtain with the linear advertising cost function, available upon request, are similar: prices are increasing in r, with $p_1^* > p_2^*$; advertising efforts fall in r, with $a_2^* > a_1^*$; firm 1's profits decrease, while firm 2's profits increase in r; consumer surplus, the number of searches and welfare all decline in r.

There are various ways to circumvent this complication. The simplest is to assume that, upon visiting a firm, a consumer can learn its type.[20] We compute our equilibrium under this assumption. Another possibility is to specify a set of beliefs after disequilibrium moves that sustain a given equilibrium. Denote with (a_1^*, p_1^*) and (a_2^*, p_2^*) the equilibrium strategy profile of the firms and suppose $p_1^* < p_2^*$. Define $\overline{p} \in (p_1^*, p_2^*)$ and let us assume that a defection to a price less than or at $\overline{p}$ is taken to come from the low-price firm, while a defection to a price above $\overline{p}$ is interpreted as coming from the high-price firm. By choosing $\overline{p}$ adequately, this system of beliefs sustains our equilibrium below.[21]

4.2 Analysis

Assume that, upon visiting a firm, a consumer learns its type. Let $\omega \in \{1, 2, 12, 21\}$ denote which firms a particular consumer visits, and in what order. Thus $\omega = 12$ implies that the consumer has first visited firm 1, and then firm 2. Let q_i^ω denote total demand for firm i from such consumers. Thus q_1^{12} denotes demand for firm 1 from consumers that visit firm 1 and 2 in that order, while q_1^1 denotes demand for firm 1 from consumers that only visit firm 1. Denote with (a_1^*, p_1^*) and (a_2^*, p_2^*) the equilibrium strategy profile of the firms. To study whether (a_i^*, p_i^*) is a best reply to (a_j^*, p_j^*), with $i \in \{1, 2\}$, $j \neq i$, we allow firm i to defect to some (a_i, p_i). Its profits then equal

$$\pi_i = p_i \left(q_i^i + q_i^{ij} + q_i^{ji} \right) - \phi_i(a_i), \tag{10}$$

where we have suppressed the arguments of the demand functions for ease of exposition. Using the derivations in Section 3.1, we can easily compute q_i^ω. Consider first q_i^i. If a buyer approaches i in her first search, the probability that this consumer immediately buys from firm i equals[22] $\Pr[x_i > \hat{x}]$, where $x_i \equiv \varepsilon_i - (p_i - p_j^*)$. With a uniform distribution, we obtain

$$q_i^i = \frac{a_i}{a_i + a_j^*} \left(1 - \hat{x} - p_i + p_j^* \right). \tag{11}$$

Conditional on visiting i first, the probability a consumer decides to also visit j only to find that j provides her with a worse deal than i is $\Pr[x_i < \hat{x} \text{ and } \varepsilon_i - p_i > \varepsilon_j - p_j^* \text{ and } \varepsilon_i > p_i]$. Hence

$$q_i^{ij} = \frac{a_i}{a_i + a_j^*} \int_{p_i}^{\hat{x} + p_i - p_j^*} (\varepsilon_i - p_i + p_j^*) d\varepsilon_i. \tag{12}$$

[20] For example, from observing the lay-out and the colors in the store, she may realize that she has actually seen more ads from the other store and hence this store must be the one with the more costly advertising technology.

[21] We thank an anonymous referee for suggesting this system of beliefs.

[22] Note that again, we must have $\hat{x} > p_2^*$, which implies that this probability is well-defined. We also assume that $\hat{x} + p_i - p_j^* \in (0, 1)$. In equilibrium, this is indeed the case.

Consider a consumer that visits j first. The probability she also visits i is $\Pr[x_j < \hat{x}]$, where $x_j \equiv \varepsilon_j - p_j^* + p_i^*$. Therefore, conditional on visiting j first, the probability that a consumer buys from i is $\Pr[x_j < \hat{x}$ and $\varepsilon_i - p_i > \varepsilon_j - p_j^*$ and $\varepsilon_i > p_i]$. This implies

$$q_i^{ji} = \frac{a_j^*}{a_i + a_j^*}\left((\hat{x} + p_j^* - p_i^*)\,(1 - \hat{x} - p_i + p_i^*) + \int_{p_i}^{\hat{x}+p_i-p_i^*}(\varepsilon_i - p_i + p_j^*)d\varepsilon_i\right) \tag{13}$$

Plugging (11), (12) and (13) into profits (10), we obtain:

$$\pi_i = p_i\frac{a_i}{a_i + a_j^*}\left(1 - \hat{x} - p_i + p_j^* + \frac{1}{2}(\hat{x}^2 - p_j^{*2})\right)$$
$$+ p_i\frac{a_j^*}{a_i + a_j^*}\left((\hat{x} + p_j^* - p_i^*)(1 - \hat{x} - p_i + p_i^*) + \frac{1}{2}(\hat{x} - p_i^*)(\hat{x} + 2p_j^* - p_i^*)\right) - \phi_i(a_i).$$

Taking the FOCs with respect to own advertising intensity and price, imposing $p_i = p_i^*$ and $a_i = a_i^*$, and doing so for $i = 1, 2$ and $j \neq i$ yields four nonlinear equalities that can be solved to find equilibrium advertising levels and prices.[23] From these FOCs, we can prove the following results:

Proposition 4 *With 2 firms, a uniform distribution of matching values, and asymmetric advertising technologies, we have that the firm that advertises more sets a lower price: $a_i^* > a_j^*$ necessarily implies $p_i^* < p_j^*$;*

This is a surprising result. We find that the effect of advertising is not to raise prices, as is commonly the case with persuasive advertising, but rather to lower them. A firm that advertises more intensively attracts a broader range of consumers that on average are less interested in the product of this particular firm. Hence this firm is inclined to charge a lower

[23] The equilibrium obtained from these FOCs can be sustained under the system of beliefs put forward above. With $\overline{p} \in (p_1^*, p_2^*)$ small defections always entail the correct interpretation. Consider now large defections. We first note that firm 1 does not have an incentive to deviate to a price above $\overline{p}$. Note that p_1^* maximizes the payoff of firm 1 under the assumption that consumers learn the type of a deviant after visiting it. By jumping up above $\overline{p}$, firm 1 triggers less favorable beliefs so this deviation can only lower its profits. On the contrary, firm 2 may have an incentive to deviate to a price below $\overline{p}$ because by jumping down to a price below $\overline{p}$ this firm triggers more favorable beliefs, which discourage consumer search. Using similar arguments as above, the payoff from such a deviation would be

$$\pi_i = p^d\frac{a^d}{a^d + a_1^*}\left(1 - \hat{x} - p^d + p_2^* + (\hat{x} - p_2^*)p_1^* + \frac{1}{2}(\hat{x} - p_2^*)^2\right)$$
$$+ p^d\frac{a_1^*}{a^d + a_1^*}\left((\hat{x} + p_1^* - p_2^*)(1 - \hat{x} - p^d + p_2^*) + \frac{1}{2}(\hat{x} - p_2^*)(\hat{x} + 2p_1^* - p_2^*)\right) - \phi_2(a^d).$$

where (a^d, p^d) denotes the deviating strategy and p_1^*, p_2^* the equilibrium prices. For every s, we can plug the equilibrium values for p_1^*, p_2^* and a_1^*, derive the most profitable deviation, check whether it entails a price lower than $\overline{p}$, and verify whether it is profitable. We have done this numerically for the linear advertising cost function. It turns out that for every s we can choose a price $\overline{p} = p^*$ (the SNE of Proposition 1) that sustains the equilibrium obtained from the FOCs no matter α.

price to try to convince these consumers to actually buy his product once they have entered his store.

By choosing to visit a second firm, consumers reveal that they do no particularly like the product the first firm offered. Hence, such consumers are less price-sensitive than consumers who still have the option to visit another shop. The firm with less advertising has a higher share of these less price-sensitive consumers. Therefore, it finds it profitable to charge a higher price. This result is in line with the study of Armstrong et al. (2009) on prominence. In their paper the prominent firm is visited first for sure. This corresponds to the case in our model where $\alpha_1 \to \infty$ and $\alpha_2 \to 0$.[24]

To see which firm advertises more, we need to put additional structure on the model. Assume that advertising technologies are linear, so $\phi_i(a) = \alpha_i a$, with $\alpha_1 < \alpha_2$. Then:[25]

Proposition 5 *With 2 firms, a uniform distribution of matching values, and linear asymmetric advertising technologies, in equilibrium, the more advertising-efficient firm will advertise more.*

4.3 Numerical analysis

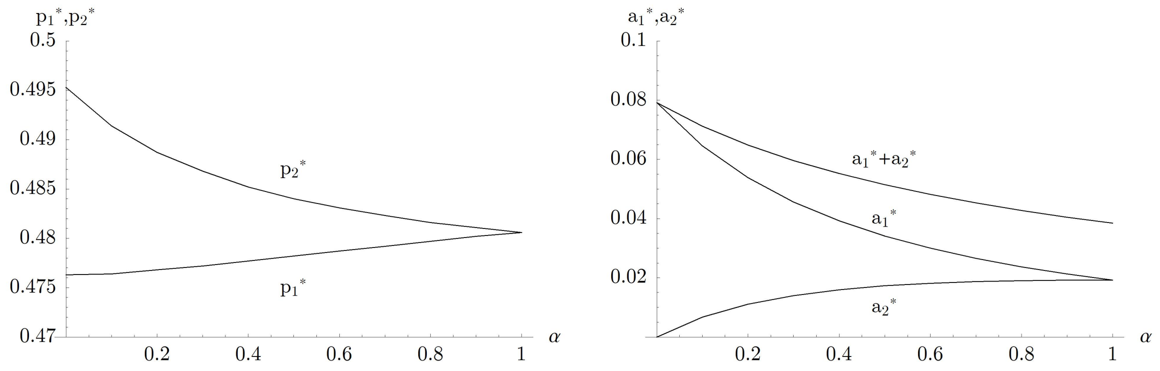
To perform comparative statics, we have to resort to a numerical analysis. We again assume linear advertising technologies. Without loss of generality, we assume that firm 1 has a more efficient advertising technology, and normalize α_2 to 1, so $\alpha_1 \leq \alpha_2 = 1$. From the analysis above in Section 4.2 we know that this implies that $a_1^* \geq a_2^*$ and $p_1^* \leq p_2^*$. The parameter $\alpha \equiv \alpha_1$ now reflects the extent of asymmetry between advertising technologies: as α increases, advertising technologies become more symmetric.

In Figure 3 we depict equilibrium prices and advertising levels as a function of α.

Remark 1 *With 2 firms, a uniform distribution of matching values, linear advertising technologies, and firm 1 the more advertising-efficient firm, we have the following:*

[24] If advertising for attention involved significant fixed-costs, asymmetric equilibria might arise. A natural equilibrium candidate would be one where a firm would pay the fixed-cost and the other firm would refrain from advertising altogether. In that case, the advertising firm would be visited first for sure and then our model would resemble that in Armstrong et al. (2009).

[25] With linear advertising costs, it is not guaranteed that our profit functions are globally well-behaved (cf. Proposition 1). To make sure that the solution to the FOCs in indeed a Nash equilibrium, in the online Technical Appendix we check numerically that any deviation from the solution to the FOCs yields a lower payoff.



(a) Price

(b) Advertising intensity

Figure 3: Price, ad intensity and firm asymmetry ($\mu = 1$, $f = 1$, $\phi_i(a) = \alpha_i a$, $s = 0.08$)

1. *if we denote by* (a_s^*, p_s^*) *equilibrium advertising levels and prices in case of symmetric advertising technologies, then* $p_1^* < p_s^* < p_2^*$.

2. *an increase in the asymmetry in firm advertising efficiency has the following effects:*

   (a) *the price of the cheapest firm decreases, that of the most expensive firm increases, while average prices also increase;*

   (b) *the advertising level of the cheapest firm increases, that of the most expensive firm decreases, while average advertising levels also increase.*

The first remark confirms the intuition behind Proposition 4: the cheaper firm also charges a lower price than what it charges with equal advertising, while the more expensive firm charges a price that is also higher than what it charges with equal advertising. Remark 2a shows that the price gap becomes more pronounced as the difference in equilibrium advertising levels increases. Remark 2b implies that, as the asymmetry in firm advertising costs increases, the difference in advertising efforts also increases.

Note that a firm that advertises more is more likely to be visited first by a consumer. As she knows that this firm charges a lower price than the other firm, she is also less likely to walk away from this firm. This suggests that the number of equilibrium searches will be lower when there is more asymmetry between advertising levels of the two firms. The following remark establishes that this is indeed the case.[26]

Remark 2 *With 2 firms, a uniform distribution of matching values, and linear advertising technologies, we have that the number of searches, and hence total search costs incurred by consumers, decreases as the asymmetry in advertising levels increases.*

Hence, advertising now has social value as it helps consumers to channel their first-visits towards better deals.

4.4 Welfare

Consumer welfare will depend on where a consumer buys, and which firms she visits. In Figure 4, we have depicted this in the $(\varepsilon_1, \varepsilon_2)$-space. The left-hand panel gives the analysis for consumers that first visit firm 1, the right-hand panel reflects consumers that first visit firm 2. In the left-hand panel, the dark-shaded area reflects the consumers that immediately buy from 1. Consumers in the vertically dashed area also buy from 1 – but only after having visited both firms. Consumers in the horizontally dashed area buy from 2 after having visited both firms. Consumers in the white bottom-left corner do not buy at all.

In the right-hand panel, consumers in the vertically dashed area again buy from 1, and consumers in the horizontally dashed area from 2, both after having visited both firms. Consumers in the lightly shaded area buy from 2, consumers in the white area do not buy at all.

Consider an increase in advertising asymmetry. The first effect of this is that total advertising of firm 1 increases, hence the left-hand panel of firm 1 will become relevant for more

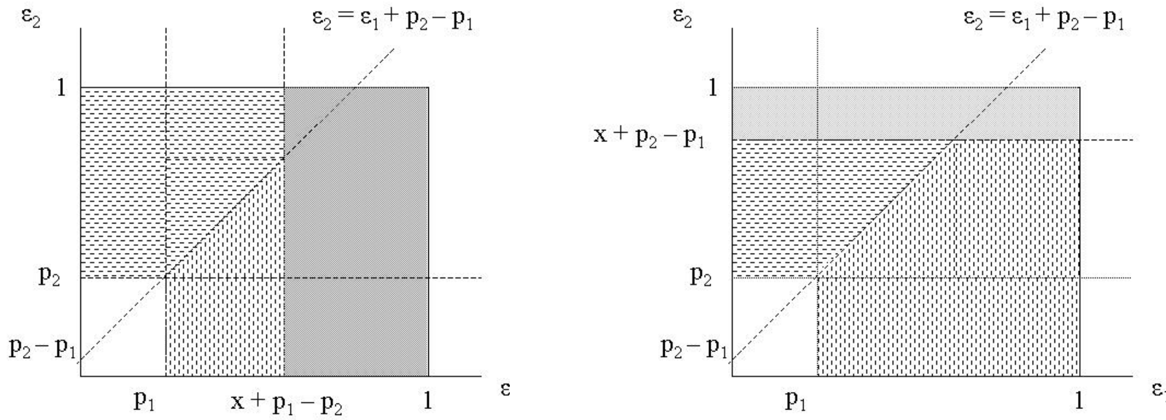
[26] If we take the results in Remark 1 as given, we can also establish this formally. By construction, each consumer searches at least once for sure. If she visits i first, the probability of a second search is $F\left(\hat{x} + p_i^* - p_j^*\right)$. If she visits j first, the probability of a second search is $F\left(\hat{x} + p_j^* - p_i^*\right)$. Denote $\gamma \equiv a_i^* / \left(a_i^* + a_j^*\right)$. We can write the expected number of searches as

$$\begin{aligned} E(\text{searches}) &= 1 + \gamma\left(\hat{x} + p_i^* - p_j^*\right) + (1-\gamma)\left(\hat{x} + p_j^* - p_i^*\right) \\ &= 1 + \hat{x} + (1-2\gamma)\left(p_j^* - p_i^*\right) \end{aligned}$$

The results in Proposition 1 imply that $\partial p_2^*/\partial\gamma > 0$ and $\partial p_1^*/\partial\gamma < 0$, so

$$\frac{\partial E(\text{searches})}{\partial \gamma} = -2\left(p_j^* - p_i^*\right) + (1-2\gamma)\left(\frac{\partial p_j^*}{\partial \gamma} - \frac{\partial p_i^*}{\partial \gamma}\right) < 0.$$

Hence the number of searches decreases as γ increases, that is, if the asymmetry between equilibrium advertising levels increases.



(a) First visit firm 1

(b) First visit firm 2

Figure 4: Consumer purchasing behavior

consumers. This is beneficial for consumer welfare, as more consumers now visit the cheaper firm first. Next, p_1^* decreases while p_2^* increases. This implies for the left-hand panel that the lines $\varepsilon_1 = p_1^*$ and $\varepsilon_1 = \hat{x} + p_1^* - p_2^*$ move to the left, while the lines $\varepsilon_2 = \varepsilon_1 + p_2^* - p_1^*$ and $\varepsilon_1 = p_2^*$ move upwards. Consumers that already bought from 1, or switch their choice to 1, benefit. The total number of searches decreases. Consumers that still buy from 2, however, are hurt, while numerical simulations show that the total number of non-buyers also increases. The effects in the right-hand panel are similar.

In sum, as asymmetry between advertising technologies increases (so α_1 decreases), the price of 1 decreases while that of 2 increases. The first effect is good news for consumers, also as they visit 1 more frequently than 2. However, as 2's price is higher, consumers who fail to find a satisfactory product at 1 are forced to accept a (much) higher price at firm 2. On average consumers search less, which lowers their search costs but also makes them less exposed to variety. The total number of consumers who buy decreases, which is obviously a source of inefficiency. The aggregate effect on consumer welfare is therefore complex.

To calculate consumer surplus, we use the same notation as above: we let $\omega \in \{1, 2, 12, 21\}$ denote which firms a consumer has visited, and in what order. Let CS_i^ω denote the total surplus of such consumers who buy from firm i. Consider, for example, consumers that buy from i that have only visited i. These consumers each incur total search costs s. Their net surplus thus is $\varepsilon_i - p_i^* - s$. Moreover, they have $\varepsilon_i > \hat{x} - p_j^* + p_i^*$. Hence

$$CS_i^i = \int_0^1 \int_{\hat{x}-p_j^*+p_i^*}^1 (\varepsilon_i - p_i^* - s) d\varepsilon_i d\varepsilon_j$$

Similarly,

$$CS_i^{ij} = \int_{p_i^*}^{\hat{x}-p_j^*+p_i^*} \int_0^{\varepsilon_i+p_j^*-p_i^*} (\varepsilon_i - p_i^* - 2s) d\varepsilon_j d\varepsilon_i$$

$$CS_j^{ij} = \int_{\hat{x}}^1 \int_0^{\hat{x}-p_j^*+p_i^*} (\varepsilon_j - p_j^* - 2s) d\varepsilon_i d\varepsilon_j + \int_{p_j^*}^{\hat{x}} \int_0^{\varepsilon_j-p_j^*+p_i^*} (\varepsilon_j - p_j^* - 2s) d\varepsilon_i d\varepsilon_j$$

Some consumers do not buy at all. The mass of these consumers is $p_1^* p_2^*$. By construction, they have visited both firms. Hence, their consumer surplus equals

$$CS_\emptyset^{ij} = -2p_1^* p_2^* s$$

Ex-ante expected consumer surplus then equals:

$$CS = \frac{a_1^*}{a_1^* + a_2}(CS_1^1 + CS_1^{12} + CS_2^{12}) + \frac{a_2^*}{a_1^* + a_2}(CS_2^2 + CS_2^{21} + CS_1^{21}) + CS_\emptyset^{ij}$$

and total welfare is $W = CS + \Pi_1^* + \Pi_2^*$ as usual.

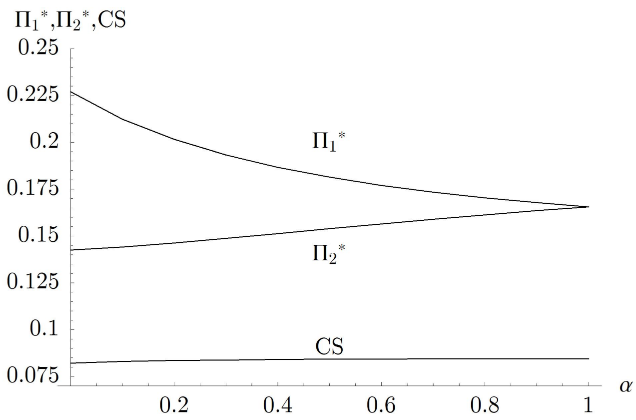


Figure 5: Welfare and firm asymmetry ($\mu = 1$, $f = 1$, $\phi_i(a) = \alpha_i a$, $s = 0.08$)

To fully appreciate the effect of a change in α on welfare, we have to resort to numerical analysis. In figure 5, we depict the components of total welfare, equilibrium profits of firms 1 and 2 and consumer surplus, as a function of α, for the case that $s = 0.08$. For different levels of s, the picture looks qualitatively the same. Profits of firm 1 decrease as firms become more symmetric, while profits of firm 2 increase – but less so. Total profits thus increase as firm 1's cost of advertising falls. The figure also shows that consumer welfare decreases after α goes down, though this effect is very small.[27] Taking together these effects, it turns out that total welfare goes up as firms become more asymmetric. This is clearly driven by the decrease in advertising outlays.[28] This result contrasts with Armstrong et al. (2009) where it is shown that if a firm becomes prominent and is always visited first a welfare loss obtains.

Simulations show that the comparative statics with respect to search costs are qualitatively unaffected by the asymmetry of advertising technologies.[29] For given advertising asymmetry α, total advertising is still increasing in search costs s. Equilibrium advertising levels of both firms increase in s, as do prices. Profits of firm 1, the most advertising efficient firm, are non-monotonic in s: initially they increase, but for high enough s they decrease. The same is true for firm 2. Total welfare decreases in search costs, as does consumer welfare.

5 Conclusion

Firms engage themselves in a battle for attention in an attempt to being visited as early as possible in the course of search of a consumer. Through investments in more appealing advertising, a firm can achieve a salient place in consumer memories. Consumers will then visit this firm sooner than the rival firms. We modelled this idea in the framework of a model of search with differentiated products. In such a framework, advertising is not a winner-takes-all contest: after a consumer has visited a firm, she may still decide to go to a different one if she does not sufficiently like the product of the current particular firm.

We found that prices and advertising levels are increasing in consumers' search costs. Yet, the effect on profits is ambiguous. If search costs are small to start with, then firms are better off if search costs increase. Instead, when search costs are already high a further increase in search costs may lower firm profits. In the latter case, getting the attention of a consumer becomes so important that firms over-dissipate the rents generated by being visited earlier than rival firms. This highlights the importance of looking at the interaction of advertising and search costs, rather than only looking at search costs or advertising in isolation. We believe this to be a general phenomenon, that applies beyond the scope of this particular model.

Another interesting finding is that firms with more efficient advertising technologies adver-

[27]For this particular parametrization, total consumer welfare falls by less than 1% as α changes from 1 to 0.1. If we set search costs equal to 0.02 rather than 0.08, exactly the same is true.

[28]The welfare result is not only driven by the fact that firm 1 has access to a more efficient advertising technology. In fact we obtain also a welfare gain if we introduce asymmetries by increasing rather than decreasing the marginal cost of advertising of firm 1. In that case too, total advertising expenditures go down and this leads to an increase in welfare.

[29]Details are available from the authors upon request.

tise more, charge lower prices and obtain greater profits than less efficient rivals. Moreover, an increase in advertising cost asymmetries leads to a fall in consumer surplus. Even though advertising serves to direct consumers to better deals on average, less advertising-efficient rivals increase their prices by so much that ultimately fewer consumers purchase a product in the market equilibrium. Asymmetries in advertising cost weaken the advertising competition between firms. This cut in advertising outlays outweigh the loss in consumer surplus and industry welfare increases.[30]

Traditionally, persuasive advertising has been modelled as advertising that increases a consumer's utility from buying the product. This interpretation is problematic, as it makes it difficult to perform welfare analysis (see Bagwell, 2007). By combining saliency enhancing advertising and search costs, our modelling approach may provide a natural way to think of persuasive advertising. In our model, advertising also increases demand for a product that is heavily advertised; however, this is not because consumers derive higher utility from advertised products but simply because they are more likely to visit shops for which they see many ads earlier than other shops, and, hence, because search costs are non-negligible, they are also more likely to buy from such shops. This difference has an implication on the relationship between prices and advertising outlays. Our model predicts that a firm that has more persuasive advertising than its competitor, will charge a lower price, as opposed to earlier work. Intuitively, consumers that only visit a shop because of its persuasive ads will be more price elastic than consumers that were already interested in the shop without seeing the ads. This price effect vanishes if both firms have the same level of persuasive advertising.

[30] If we consider efforts in advertising as mere transfers to the advertising industry, then total welfare would be higher if advertising were banned.

Appendix

Proof of Proposition 1. As mentioned in the main text, this proof has four steps.

Step 1. We first derive the expressions (6) and (7) given in the proposition. Maximizing firm i's profits in equation (5) with respect to a_i and p_i yields the following FOCs:

$$p_i \sum_{k=1}^{n} \frac{\partial \lambda_k^i(a_i, p_i; a^*, p^*)}{\partial a_i} - \phi'(a_i) = 0 \tag{14}$$

$$\sum_{k=1}^{n} \lambda_k^i(a_i, p_i; a^*, p^*) + R(p_i; p^*) + p_i \left[\sum_{k=1}^{n} \frac{\partial \lambda_k^i(a_i, p_i; a^*, p^*)}{\partial p_i} + \frac{\partial R(p_i; p^*)}{\partial p_i} \right] = 0. \tag{15}$$

Using the expression for λ_k^i in equation (3) and applying symmetry we have

$$\begin{aligned}
\frac{\partial \lambda_1^i}{\partial a_i} &= \frac{n-1}{n^2 a^*} (1 - F(\hat{x})) \\
&\cdots \\
\frac{\partial \lambda_k^i}{\partial a_i} &= \left[\frac{1}{n-k+1} \sum_{\ell=1}^{k-1} \left[\frac{-(n-\ell)}{(n-\ell+1)^2 a^*} \prod_{m \neq \ell}^{k-1} \frac{n-m}{n-m+1} \right] \right. \\
&\quad \left. + \frac{n-i}{(n-i+1)^2 a^*} \prod_{\ell=1}^{k-1} \frac{n-\ell}{n-\ell+1} \right] F(\hat{x})^{k-1} (1 - F(\hat{x})) \\
&\cdots \\
\frac{\partial \lambda_n^i}{\partial a_i} &= \sum_{\ell=1}^{n-1} \left[\frac{-(n-\ell)}{(n-\ell+1)^2 a^*} \prod_{m \neq \ell}^{n-1} \frac{n-m}{n-m+1} \right] F(\hat{x})^{n-1} (1 - F(\hat{x})).
\end{aligned}$$

Note that

$$\prod_{\ell=1}^{k-1} \frac{n-\ell}{n+1-\ell} = \frac{n-1}{n} \cdot \frac{n-2}{n-1} \cdot \ldots \cdot \frac{n+1-k}{n+2-k} = \frac{n+1-k}{n},$$

which implies that

$$\frac{\partial \lambda_k^i}{\partial a_i} = \frac{1}{na^*} \left[\frac{n-k}{n-k+1} - \sum_{\ell=1}^{k-1} \frac{1}{(n-\ell+1)} \right] F(\hat{x})^{k-1} (1 - F(\hat{x})).$$

Moreover, we note that $\sum_{k=1}^{n} \lambda_k(a^*, p^*) = \frac{1}{n}(1 - F(\hat{x})^n)$.

Using these derivations and the expression for $R(p^*)$ in equation (4), the FOCs (14) and (15) can be rewritten as:

$$p^* \sum_{k=1}^{n} \frac{1}{na^*} \left(\frac{n-k}{n-k+1} - \sum_{\ell=1}^{k-1} \frac{1}{(n-\ell+1)} \right) F(\hat{x})^{k-1} (1 - F(\hat{x})) - \phi'(a^*) = 0, \tag{16}$$

$$\frac{1-F(\hat{x})^n}{n} + \int_{\frac{p^*}{\mu}}^{\hat{x}} F(\varepsilon)^{n-1} f(\varepsilon) d\varepsilon - \frac{p^*}{\mu} \frac{f(\hat{x})}{n} \frac{1-F(\hat{x})^n}{1-F(\hat{x})} -$$

$$\frac{p^*}{\mu} \left(\int_{\frac{p^*}{\mu}}^{\hat{x}} (n-1) F(\varepsilon)^{n-2} f(\varepsilon)^2 d\varepsilon + F\left(\frac{p^*}{\mu}\right)^{n-1} f\left(\frac{p^*}{\mu}\right) - F(\hat{x})^{n-1} f(\hat{x}) \right) = 0. \quad (17)$$

It is now readily seen that using the integration by parts formula in (17) yields (7).

To see that equation (16) implies (6), denote

$$C_k \equiv \frac{n-k}{n-k+1} - \sum_{\ell=1}^{k-1} \frac{1}{n-\ell+1}, \quad (18)$$

so we can rewrite (16) as

$$a^* \phi'(a^*) = \frac{p^*}{n} (1 - F(\hat{x})) \sum_{k=1}^{n} C_k \cdot F(\hat{x})^{k-1}. \quad (19)$$

Note that $C_k - C_{k-1} = -1/(n-k+1)$ and that from (18) we have $C_1 = (n-1)/n$. Therefore, by induction,

$$C_k = \frac{n-1}{n} - \sum_{\ell=1}^{k-1} \frac{1}{n-\ell}.$$

Plugging this back into (19), we obtain

$$\begin{aligned} a^* \phi'(a^*) &= \frac{p^*}{n} (1 - F(\hat{x})) \left[\frac{n-1}{n} \sum_{k=0}^{n-1} F(\hat{x})^k - \sum_{\ell=1}^{n-1} \left(\frac{1}{n-\ell} \sum_{k=\ell}^{n-1} F(\hat{x})^k \right) \right] \\ &= \frac{p^*}{n} \left[\frac{n-1}{n} \cdot (1 - F(\hat{x})^n) - \sum_{\ell=1}^{n-1} \left(\frac{1}{n-\ell} \right) F(\hat{x})^\ell \left(1 - F(\hat{x})^{n-\ell} \right) \right]. \end{aligned}$$

which can be further simplified to (6).

Step 2. We now show that there exists a pair (p^*, a^*) that satisfies the system of equations (6) and (7). By inspection of (6), it is immediately clear that for any p^* there is a unique a^* that accompanies p^*. To see that such a^* is non-negative, consider the expression

$$A \equiv 1 - F(\hat{x})^n - \sum_{k=0}^{n-1} \frac{F(\hat{x})^k \left(1 - F(\hat{x})^{n-k}\right)}{n-k} \quad (20)$$

and notice that

$$\frac{dA}{d\hat{x}} = -\sum_{k=1}^{n-1} \frac{k F(\hat{x})^{k-1} \left(1 - F(\hat{x})^{n-k}\right)}{n-k} f(\hat{x}) < 0. \quad (21)$$

Therefore, A is monotonically decreasing in $\hat{x}$ and since $\lim_{\hat{x}\to 1} A = 0$ we conclude a^* is non-negative.

Consider now equation (7). To study the existence of a solution in p^*, it is useful to rewrite it as follows:

$$\frac{1-F(p^*/\mu)^n}{np^*/\mu} = \frac{f(\hat{x})}{n}\frac{1-F(\hat{x})^n}{1-F(\hat{x})} - \int_{p^*/\mu}^{\hat{x}} F(\varepsilon)^{n-1} f'(\varepsilon) d\varepsilon. \tag{22}$$

Note that the RHS of (22) is finite when $p^* \to 0$. The LHS is a positive-valued function that decreases monotonically in p^*. Moreover, when $p^* \to 0$ the LHS goes to ∞. Hence, for $p^* \to 0$ the LHS is larger than the RHS. If $p^* \to \mu\hat{x}$, the LHS is smaller than the RHS if and only if $1-F(\hat{x}) < \hat{x}f(\hat{x})$. Since $\hat{x} > p^m/\mu > p^*/\mu$ and by definition $1-F(p^m/\mu)-(p^m/\mu)f(p^m/\mu) = 0$, logconcavity of monopoly profits implies that this condition always holds. With the LHS larger that the RHS at $p^* \to 0$, but smaller at $p^* \to \mu\hat{x}$, continuity implies that there must be at least one $p^* \in (0, \mu\hat{x})$ such that (22) is satisfied. If we assume also that $f' \geq 0$, we have that the RHS is strictly increasing in p^*, in which case (22) has a unique solution.

Step 3. To make sure that the solution to the FOCs (6) and (7) is a SNE, the payoff function of a firm i must be globally quasi-concave on its domain. The next three claims, proven in an online Appendix, show this. Let D denote the domain of firm i's payoff function. Note that $D \equiv \{(a_i, p_i) \in [0, \infty) \times (0, p^m)\}$ and consider the following split of the domain $D = D_1 \cup D_2 \cup D_3$ where $D_1 \equiv \{(a_i, p_i) \in (0, \infty) \times (0, \mu F^{-1}(1) - \mu\hat{x} + p^*)\}$, $D_2 = \{(a_i, p_i) \in [0, \infty) \times [\mu F^{-1}(1) - \mu\hat{x} + p^*, p^m)\}$ and $D_3 \equiv \{(a_i, p_i) \in \{0\} \times (0, p^m\}$. On D_1, $\Pi_i(a_i, p_i; a^*, p^*)$, is given in (5).[31]

Claim 1 *On D_1, the function $\Pi_i(a_i, p_i; a^*, p^*)$ is strictly concave in a_i.*

Claim 2 *On D_1, $\Pi_i(a_i, p_i; a^*, p^*)$ is not necessarily quasi-concave in p_i. However, when F represents the uniform distribution, then $\Pi_i(a_i, p_i; a^*, p^*)$ is strictly concave in p_i.*

Claim 3 *When F is the uniform distribution, and when ϕ'' is sufficiently large, the function $\Pi_i(a_i, p_i; a^*, p^*)$ is globally strictly concave on D_1.*

Claims 1, 2 and 3 together imply that there does not exist any profitable deviation from (a^*, p^*) in the set D_1 provided that matching values are uniformly distributed and the advertising cost function is sufficiently convex. To complete the proof, we now study deviations outside the set D_1.

Step 4. Consider now deviations to pairs (a_i, p_i) in the sets D_2 and D_3 defined above, i.e., we need to make sure that a firm i has no interest in deviating by charging a price such

[31] Deviations for which $p_i \geq \mu F^{-1}(1) - \mu\hat{x} + p^*$ are special because in those situations firm i would only sell to consumers who have walked away from all other rivals; we treat these cases later in step 4.

that $1 - F(\hat{x} + \Delta) = 0$. In that case no consumer would ever stop searching at firm i and the deviant firm would only sell to the consumers who happen to find no acceptable product elsewhere. Deviating profits for this situation would be

$$\Pi_i(a_i, p_i; a^*, p^*) = p_i \int_{p_i/\mu}^{1} F(\varepsilon - \Delta)^{n-1} f(\varepsilon) d\varepsilon - \phi(a_i). \tag{23}$$

By monotonicity of this payoff, it is clear that the deviant firm would find it optimal to accompany the deviating price with an advertising effort equal to zero.[32] Because of log-concavity of f, this profits expression is quasi-concave in p_i (see Caplin and Nalebuff, 1991). Taking the derivative with respect to p_i and setting $p_i = p^*$ yields:

$$\frac{1 - F(p^*/\mu)^n}{n} - \frac{p^*}{\mu}\left(f(1) - \int_{p^*/\mu}^{1} F(\varepsilon)^{n-1} f'(\varepsilon) d\varepsilon\right). \tag{24}$$

where the last equality follows from integration by parts. This expression is exactly the limit of the FOC in (7) when $\hat{x} \to 1$. We will show later in the proof of Proposition 2 that the expression

$$\frac{f(\hat{x})}{n} \frac{1 - F(\hat{x})^n}{1 - F(\hat{x})} - \int_{p^*/\mu}^{\hat{x}} F(\varepsilon)^{n-1} f'(\varepsilon) d\varepsilon. \tag{25}$$

is increasing in $\hat{x}$. This implies that (24) is negative and therefore the profits expression in (23) is decreasing at $p_i = p^*$. This fact along with the quasi-concavity of the expression in (23) implies that deviating profits are monotonically decreasing in p_i, for all $p_i \in [\tilde{p_i}, p^m]$, where $\tilde{p_i}$ is the solution to $1 - F(\hat{x} + (\tilde{p_i} - p^*)/\mu) = 0$. As a result, deviating to a price above p^* is not profitable.

Taken together, steps 1, 2, 3 and 4 establish the proposition. ■

Proof of Proposition 2. 1. The price result follows straightforwardly from the equilibrium condition (7), which does not depend on advertising costs. For the advertising result, consider the equilibrium condition (6) and note that a change in advertising costs should leave the product $a^*\phi'(a^*)$ constant. Consider two advertising cost functions ϕ_1 and ϕ_2, with $\phi_1'(a) > \phi_2'(a)$ for all a. Equilibrium requires $a_1^*\phi_1'(a_1^*) = a_2^*\phi_2'(a_2^*)$. As $\phi_1' > \phi_2'$, we require $a_1^*\phi_1'(a_1^*) < a_2^*\phi_1'(a_2^*)$. Convexity of ϕ_1 implies that $a\phi_1'(a)$ is strictly increasing in a, hence equilibrium requires $a_1^* < a_2^*$.

2. We start with the price result. Building on the proof of Proposition 1, the equilibrium price is given by the solution to (22). In this equation the effects of higher search costs are

[32] Likewise, notice that if the deviating firm sets an advertising effort equal to zero, the firm would be visited last and its profit would be similar to that in (23).

manifested only through changes in $\hat{x}$. The LHS of (22) decreases in p^* and does not depend on $\hat{x}$. The RHS is nondecreasing in p^* for any distribution that has $f' \geq 0$. Taking the derivative of the RHS of (22) with respect to $\hat{x}$ yields:

$$\frac{\left[f'(\hat{x})(1-F(\hat{x})^n) - nF(\hat{x})^{n-1}f^2(\hat{x})\right](1-F(\hat{x})) + f(\hat{x})^2(1-F(\hat{x})^n)}{n(1-F(\hat{x}))^2} - F(\hat{x})^{n-1}f'(\hat{x}) \quad (26)$$

Collecting terms the expression in (26) can be rewritten as:

$$\frac{1}{n}\left(f'(\hat{x}) + \frac{f^2(\hat{x})}{(1-F(\hat{x}))}\right)\left[\frac{1-F(\hat{x})^n}{1-F(\hat{x})} - nF(\hat{x})^{n-1}\right] \quad (27)$$

The first term is positive because of log concavity of $1-F$. The second term is also positive because it equals $\sum_{k=0}^{n-1}\left[F(\hat{x})^k - F(\hat{x})^{n-1}\right]$ and F is a distribution function. We thus have that the RHS of (22) is increasing in $\hat{x}$ and therefore decreasing in s.

We now take the advertising result. Rewrite a^* as

$$a^*\phi'(a^*) = \frac{p^*A}{n}, \quad (28)$$

where A is given in equation (20). Taking the derivative of $a^*\phi'(a^*)$ with respect to $\hat{x}$ gives

$$\frac{d}{d\hat{x}}(a^*\phi'(a^*)) = \frac{A}{n}\frac{dp^*}{d\hat{x}} + \frac{p^*}{n}\frac{dA}{d\hat{x}}.$$

Since ϕ is convex, we need $a^*\phi'(a^*)$ to be decreasing in $\hat{x}$. But this is true since we know already that $dp^*/d\hat{x} < 0$ and in Proposition 1 we have shown that $dA/d\hat{x} < 0$.

3. For the price result, we apply the implicit function theorem to equation (7). Let $\Gamma(\cdot)$ denote the LHS of (7). Noting that $\hat{x}$ depends positively on μ, we have

$$\frac{dp^*}{d\mu} = -\frac{\frac{\partial\Gamma}{\partial\mu} + \frac{\partial\Gamma}{\partial\hat{x}}\frac{\partial\hat{x}}{\partial\mu}}{\frac{\partial\Gamma}{\partial p^*}} = \frac{p^*}{\mu} + \frac{-\frac{\partial\Gamma}{\partial\hat{x}}\frac{\partial\hat{x}}{\partial\mu}}{\frac{\partial\Gamma}{\partial p^*}}.$$

Note now that $-\frac{\partial\Gamma}{\partial\hat{x}}$ equals expression (27) while

$$\frac{\partial\hat{x}}{\partial\mu} = \frac{s}{\mu^2\left(1-F(\hat{x})\right)}.$$

In addition,

$$\frac{\partial\Gamma}{\partial p^*} = -\frac{nF(p^*/\mu)^{n-1}f(p^*/\mu)\left(p^*/\mu\right) + \left(1-F(p^*/\mu)^n\right)}{np^{*2}/\mu} - \frac{1}{\mu}F(p^*/\mu)^{n-1}f'(p^*/\mu).$$

Consider the case where μ is large ($\mu \to \infty$). In such a case, $\hat{x} \to 1$ and $F(\hat{x}) \to 1$ so the ratio p^*/μ is finite since it is the solution to the following equation

$$\frac{1-F(p^*/\mu)^n}{n(p^*/\mu)} - f(1) + \int_{p^*/\mu}^{1} F(\varepsilon)^{n-1}f'(\varepsilon)d\varepsilon = 0.$$

Moreover, since $\lim_{F\to 1}(1-F^n)/(1-F) = n$

$$-\lim_{\mu\to\infty}\frac{\partial\Gamma}{\partial\hat{x}} = \lim_{\mu\to\infty}\frac{1}{n}\frac{f^2(\hat{x})}{(1-F(\hat{x}))}\left[\frac{1-F(\hat{x})^n}{1-F(\hat{x})} - nF(\hat{x})^{n-1}\right].$$

Using the L´Hopital rule we obtain

$$\lim_{\mu\to\infty}\frac{\partial\hat{x}}{\partial\mu} = \lim_{\mu\to\infty}\frac{s}{\mu^2(1-F(\hat{x}))} = \lim_{\mu\to\infty}\frac{-2s/\mu^3}{-f(\hat{x})\frac{\partial\hat{x}}{\partial\mu}} = \lim_{\mu\to\infty}\frac{2(1-F(\hat{x}))}{\mu f(\hat{x})}.$$

Therefore

$$-\lim_{\mu\to\infty}\frac{\partial\Gamma}{\partial\hat{x}}\frac{\partial\hat{x}}{\partial\mu} = \lim_{\mu\to\infty}\frac{2}{n}\frac{f(\hat{x})}{\mu}\left[\frac{1-F(\hat{x})^n}{1-F(\hat{x})} - nF(\hat{x})^{n-1}\right] = 0.$$

Since

$$\lim_{\mu\to\infty}\frac{\partial\Gamma}{\partial p^*} = -\infty$$

we conclude that $\lim_{\mu\to\infty} dp^*/d\mu = \lim_{\mu\to\infty}(p^*/\mu) > 0$.

To prove the advertising result, we consider the case where $n = 2$ and F is the uniform distribution. The equilibrium price and advertising effort are given in equations (8) and (9). Notice that

$$\frac{4}{1-\hat{x}}\frac{d(a^*\phi'(a^*))}{d\mu} = -(1-\hat{x})\frac{p^*}{\mu}\left(\frac{1-\hat{x}}{2\sqrt{5+\hat{x}(2+\hat{x})}}\right) < 0$$

Since ϕ is convex, the result follows.

4. Let (a_n, p_n) be the solution to the FOCs (6) and (7) when the number of firms is n. We first prove that $a_n \to 0$ as $n\to\infty$. First note that $a_n \to 0$ if and only if $a_n\phi'(a_n)\to 0$. From equation (28), we have

$$\lim_{n\to\infty} a_n\phi'(a_n) = \lim_{n\to\infty} p_n \lim_{n\to\infty}\frac{A}{n}$$

It is easy to see that $\lim_{n\to\infty} p_n = \mu(1-F(\hat{x}))/f(\hat{x})$, which is strictly positive (Wolinsky, 1986). Therefore we need to show that $\lim_{n\to\infty} A/n = 0$. We have

$$\lim_{n\to\infty}\frac{A}{n} = -\lim_{n\to\infty}\sum_{k=0}^{n-1}\frac{F(\hat{x})^k\left(1-F(\hat{x})^{n-k}\right)}{n(n-k)} = -\lim_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n-1}\frac{1}{n-k}$$

where the last equality follows from the fact that $F(\hat{x})^k\left(1-F(\hat{x})^{n-k}\right)$ is strictly positive and bounded by 1. Consider the sum $\sum_{k=0}^{n-1}\frac{1}{n-k}$, which can be rewritten as $\sum_{k=1}^{n}\frac{1}{k}$. It is known that the Euler number γ is given by

$$\gamma \equiv \lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{k} - \ln n\right)$$

Therefore

$$\lim_{n\to\infty}\frac{A}{n} = -\lim_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n-1}\frac{1}{n-k} = -\lim_{n\to\infty}\frac{\gamma+\ln n}{n} = 0$$

For general advertising costs we have $na^* = p^* A/\phi'(a^*)$. Noting that $\lim_{n\to\infty} A = 1$ we have $\lim_{n\to\infty} na^* = \mu(1 - F(\hat{x}))/f(\hat{x})\phi'(0)$.

Finally we prove that when F is the uniform distribution, an increase in n increases a^* for n sufficiently low. Setting $n = 2$ in the FOC (6) yields $a_2\phi'(a_2) = p_2(1 - F(\hat{x}))^2/4$ while setting $n = 3$ in the same FOC yields $a_3\phi'(a_3) = p_3(1 - F(\hat{x}))^2(4 + 5F(\hat{x}))/18$. Since $a\phi'(a)$ is increasing in a, we have that $a_3 > a_2$ provided that $p_3(4+5F(\hat{x}))/9 > p_2/2$. We note first that this inequality clearly holds when F is the uniform distribution and search costs are high, i.e. $\hat{x} \to p^m/\mu$. In such a case, $F(p^m/\mu) \to 1/2$ and p_2 and p_3 both approach the monopoly price. Moreover, we note that for $n = 2$ and $n = 3$, it is possible to solve for equilibrium prices. Doing so, tedious calculations reveal that the required inequality is satisfied for the entire range of search costs. ■

Proof of Proposition 3. First note that the payoff of a firm in SNE is:

$$\Pi_i(a^*, p^*) = \frac{1}{n}p^*(1 - F(p^*/\mu)^n) - \phi(a^*)$$

We are interested in the derivative of Π_i with respect to s. Since $\hat{x}$ falls in s, we have

$$\frac{d\Pi(\cdot)}{d\hat{x}} = \frac{\partial \Pi}{\partial p^*}\frac{dp^*}{d\hat{x}} + \frac{\partial \Pi}{\partial a^*}\frac{da^*}{d\hat{x}} \tag{29}$$

Equation (29) shows that search costs do not affect profits directly but via p^* and a^*. From Proposition 2, we know that $dp^*/d\hat{x} < 0$ and $da^*/d\hat{x} < 0$. In equilibrium it is obvious that all firms gain if they all raise their prices, i.e., $\partial\Pi/\partial p^* > 0$. This implies that an increase in s tends to raise profits because p^* increases; however, since $\partial\Pi/\partial a^* = -\phi'(a^*) < 0$, an increase in s tends to lower profits because a^* goes up. As a result, an increase in s operates on profits in two ways that go in opposite directions.

1. To prove this, we first use the FOC (6), to rewrite (29) as

$$\frac{d\Pi(\cdot)}{d\hat{x}} = \left(\frac{\partial \Pi}{\partial p^*} - \phi'(a^*)\frac{\partial a^*}{\partial p^*}\right)\frac{dp^*}{d\hat{x}} - \phi'(a^*)\frac{\partial a^*}{\partial \hat{x}}. \tag{30}$$

Second we note that, from (28), we have

$$\frac{\partial a^*}{\partial p^*} = \frac{A}{n(\phi'(a^*) + a^*\phi''(a^*))}$$

Moreover, from the derivations above in Proposition 2 we have that

$$\frac{dp^*}{d\hat{x}} = \frac{-\frac{\partial\Gamma}{\partial\hat{x}}}{\frac{\partial\Gamma}{\partial p^*}} = \frac{-\frac{1}{n}\left(f'(\hat{x}) + \frac{f^2(\hat{x})}{(1-F(\hat{x}))}\right)\left[\frac{1-F(\hat{x})^n}{1-F(\hat{x})} - nF(\hat{x})^{n-1}\right]}{\frac{nF(p^*/\mu)^{n-1}f(p^*/\mu)(p^*/\mu)+(1-F(p^*/\mu)^n)}{np^{*2}/\mu} + \frac{1}{\mu}F(p^*/\mu)^{n-1}f'(p^*/\mu).} \tag{31}$$

Consider the case where search costs are very small, that is $\hat{x} \to 1$ and so $F(\hat{x}) \to 1$. In such a case, since $\lim_{F\to 1}(1-F^n)/(1-F) = n$, the numerator of (31) goes to $-f^2(1)(n-1)/2$. When $f' \geq 0$ the denominator is positive so we conclude that $\lim_{\hat{x}\to 1} dp^*/d\hat{x}$ is finite and negative. We note now that $\lim_{\hat{x}\to 1} A = 0$ so $\lim_{\hat{x}\to 1} \partial a^*/\partial p^* = 0$. As a result, the first term in the RHS of (30) is a finite negative number when $\hat{x} \to 1$.

Consider now the second term in the RHS of (30). Using (28) again, we have

$$\frac{\partial a^*}{\partial \hat{x}} = \frac{p^*(\partial A/\partial \hat{x})}{n(\phi'(a^*) + a^*\phi''(a^*))} \tag{32}$$

and plugging (21) in this equation we obtain

$$\lim_{\hat{x}\to 1} \frac{\partial a^*}{\partial \hat{x}} = \lim_{\hat{x}\to 1} \frac{p^*}{n(\phi'(a^*) + a^*\phi''(a^*))} \lim_{\hat{x}\to 1} \sum_{k=1}^{n-1} \frac{kF(\hat{x})^{k-1}\left(1 - F(\hat{x})^{n-k}\right)}{(n-k)} f(\hat{x}) = 0$$

for any increasing advertising cost function. As a result we have proven that $\lim_{\hat{x}\to 1} d\Pi(\cdot)/d\hat{x}$ equals a finite negative number. Since $\lim_{\hat{x}\to 1} d\hat{x}/ds = -\infty$, we conclude profits increase in a neighborhood of $s = 0$.

2. (i) For the family of advertising cost functions $\phi(a) = \alpha a^\gamma$ with $\alpha > 0, \gamma \geq 1$, it holds that $\phi(a) = a^*\phi'(a^*)/\gamma$ so from the equilibrium condition (6) we have $\phi(a) - \frac{p^*A}{\gamma n} = 0$. From this equality we obtain

$$-\phi'(a^*)\frac{da^*}{d\hat{x}} = -\frac{1}{\gamma n}\left(\frac{\partial p^*}{\partial \hat{x}}A + p^*\frac{\partial A}{\partial \hat{x}}\right),$$

which goes to zero as $\gamma \to \infty$. As a result, the advertising effect in (29) vanishes and therefore profits increase in s. (ii) However, profits need not be increasing in search costs. Here we provide a counter-example. Consider again the case when $n = 2$ and F is the uniform distribution. Equilibrium price and advertising are given in (8) and (9), while profits equal

$$\Pi^* = \frac{p^*}{2}\left(1 - \left(\frac{p^*}{\mu}\right)^2\right) - \phi(a^*) = \frac{p^*}{2}\left(1 - \left(\frac{p^*}{\mu}\right)^2 - \frac{(1-\hat{x})^2}{2\gamma}\right)$$

where s ranges from 0 to $\mu/8$ in this case. Setting $\mu = 1$, Figure 2 plots Π^* against s for various values of the elasticity of the advertising cost function γ. For relatively low values of γ, profits are non-monotonic in s, first increasing and then decreasing. ■

Proof of Proposition 4 Define the probability that i is visited first as γ. Thus $\gamma \equiv a_i^*/\left(a_i^* + a_j^*\right)$. After setting $p_i = p_i^*$, the FOC in prices can then be written as

$$h_1(\gamma, p_1^*, p_2^*) \equiv \gamma\left(1 - \hat{x} - 2p_1^* + p_2^* + \frac{1}{2}(\hat{x}^2 - p_2^{*2})\right)$$
$$+ (1-\gamma)\left(\frac{1}{2}(2 - \hat{x} - 3p_1^*)(\hat{x} - p_1^* + p_2^*) + \frac{1}{2}(\hat{x} - p_1^*)p_2^*\right) = 0.$$

The FOC for the other firm yields a similar condition: $h_2(1-\gamma, p_2^*, p_1^*) = 0$. This implies that equilibrium also requires that $h_1(\gamma, p_1^*, p_2^*) = h_2(1-\gamma, p_2^*, p_1^*)$. With $2\gamma > 1$ and $p_1^* > p_2^*$, it can be shown that this equality cannot be satisfied. This implies that $\gamma > 1/2$ (and thus $a_1^* > a_2^*$) necessarily requires $p_1^* < p_2^*$.■

Proof of Proposition 5 After setting $a_i = a_i^*$, the FOC in advertising levels is

$$\alpha_i = p_i^* \frac{a_j^*}{(a_i^* + a_j^*)^2} \Big[1 - \hat{x} - p_i^* + p_j^* + \frac{1}{2}(\hat{x}^2 - p_j^{*2}) \\ -(\hat{x} + p_j^* - p_i^*)(1 - \hat{x}) - \frac{1}{2}(\hat{x} - p_i^*)(\hat{x} + 2p_j^* - p_i^*) \Big]$$

Suppose that firm 1 is the more advertising-efficient firm, so $\alpha_1 < \alpha_2$. We then require that the RHS of the FOC for firm 1 is smaller than that for firm 2. It can be shown that that inequality simplifies to

$$p_1^* a_2^* < p_2^* a_1^*.$$

Suppose that $a_1^* < a_2^*$. This, by Proposition 4, necessarily implies $p_1^* > p_2^*$, hence $p_1^* a_2^* > p_2^* a_1^*$. But this contradicts the inequality derived above, thus establishing the result. ■

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